Solutions to Mock JEE MAIN – 6 (CBT) | JEE 2024

PHYSICS

SECTION-1

1.(C) $E_0 = 7.2\times5\times10^3\ V/m$; $\quad C = \dfrac{4.5\times10^{15}}{1.5\times10^7} = 3\times10^8 m/s.$

$$B_0 = \frac{E_0}{C} = \frac{7.2\times5\times10^3}{3\times10^8} = 2.4\times5\times10^{-5} Wb/m^2$$

$$\hat{B} = \hat{c}\times\hat{E} \quad \text{Now } \hat{B} = -\hat{k}\times\frac{3\hat{i}+4\hat{j}}{\sqrt{25}} \Rightarrow \hat{B} = \frac{(4\hat{i}-3\hat{j})}{5}$$

$$\vec{B} = B_0\cos(1.5\times10^7 z + 4.5\times10^{15}t)\times\hat{B}$$

$$\Rightarrow \quad \vec{B} = 2.4\times10^{-5}\cos(1.5\times10^7 z + 4.5\times10^{15}t)(4\hat{i}-3\hat{j})Wb/m^2$$

2.(A) $13.6z^2\left[\dfrac{1}{4}-\dfrac{1}{9}\right] = 68eV$

$$13.6z^2\left[\frac{5}{36}\right] = 68eV \Rightarrow z^2 = 36 \Rightarrow z = 6$$

$$K.E. = \frac{13.6Z^2}{1^2} = \frac{68\times36}{5} = 489.6eV$$

3.(C) $U_f = 2n\left(\dfrac{3R}{2}T\right) = 3nRT$; $\quad U_i = n\left(\dfrac{5R}{2}T\right) = \dfrac{5nRT}{2}$

Heat energy required is $\Rightarrow \quad \Delta U = 3nRT - \dfrac{5}{2}nRT = \dfrac{nRT}{2}$

$$\Rightarrow \quad \Delta U = \frac{2}{2}\times\frac{25}{3}\times300 \quad \Rightarrow \quad \Delta U = 2500J = 2.5\ kJ$$

4.(A) $y = 0 \Rightarrow 16t - 5t^2 \Rightarrow 5t^2 = 16t \Rightarrow t = 0,\ t = \dfrac{16}{5}s$

$T = 3.2\ s$; $\quad R = 9T = 9\times3.2 = 28.8\ m$

5.(B)

6.(B) From momentum conservation,

$$m_1v_1 = (m_1+m_2)v_2 \quad \Rightarrow \quad 500\times6 = (600)\times v_2 \quad \Rightarrow \quad v_2 = 5\,m/s$$

$$K.E._i = \frac{1}{2}m_1v_1^2 = \frac{1}{2}500\times(6)^2 = (250\times36)J$$

$$K.E._f = \frac{1}{2}(m_1+m_2)v_2^2 = \frac{1}{2}(600)(5)^2 = (25\times300)J$$

$$\Delta K.E. = 25\times300 - 250\times36 = 25(300-360) = -25\times60 = -1500\ J$$

7.(B) When input = $-5V$. Diode is R.B. and does not conduct Hence output = 0.
When input = $5V$, diode is F.B. and conducts. Hence output = $5V$.

8.(C) $\dfrac{4.5}{R} = \dfrac{30}{70}$ by balanced wheatstone bridge condition

$\Rightarrow \quad R = 4.5 \times \frac{7}{3} = \frac{31.5}{3} = 10.5\Omega \Rightarrow R = 10.5\Omega$

$\Rightarrow \quad \frac{\rho l}{A} = 10.5 \Rightarrow \frac{\rho \times 15 \times 10^{-2}}{\frac{22}{7} \times 7 \times 10^{-8}} = 10.5$

$\Rightarrow \quad \rho = \frac{10.5 \times 22}{15} \times 10^{-6} = \frac{105}{15} \times 22 \times 10^{-7} \quad \Rightarrow \quad \rho = 154 \times 10^{-7}\,\Omega - m.$

9.(B) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \quad \frac{1}{x} - \frac{1}{u} = \frac{1}{40} \quad \ldots \text{(i)}$

$\frac{1}{x-36} - \frac{1}{u} = \frac{1}{20} \quad \ldots \text{(ii)}$

$\Rightarrow \quad \frac{1}{x} - \frac{1}{40} = \frac{1}{x-36} - \frac{1}{20} \quad \Rightarrow \quad \frac{40-x}{40x} = \frac{20-(x-36)}{20(x-36)}$

$\Rightarrow \quad (40-x)(x-36) = 2x(56-x) \quad \Rightarrow \quad -1440 - x^2 + 36x + 40x = 112x - 2x^2$

$\Rightarrow \quad x^2 - 36x - 1440 = 0 \quad \Rightarrow \quad x = 60\,cm$

$\Rightarrow \quad \frac{1}{60} - \frac{1}{u} = \frac{1}{40} \Rightarrow \frac{1}{u} = \frac{1}{60} - \frac{1}{40} \quad \Rightarrow \quad \frac{1}{u} = \frac{-1}{120} \Rightarrow u = -120\,cm$

So, object distance is 120 *cm*.

10.(B)

11.(A) $\frac{R_1}{R_2} = \frac{L_1}{L_2} \times \frac{A_2}{A_1} = \frac{4}{1} \times \frac{4}{9} = \frac{16}{9}$

$\frac{R_1}{20} = \frac{16}{9} \quad \Rightarrow \quad R_1 = \frac{320}{9}\Omega$

$R_{eq} = R_1 + R_2 \quad \Rightarrow \quad 20 + \frac{320}{9} = \frac{500}{9}\Omega = R_{eq}$

12.(C) For angle of minimum deviation

$r_1 = r_2 = r = \frac{A}{2} \Rightarrow r = 37°$

$\frac{\sin i}{\sin r} = u \Rightarrow \sin i = \sin r \times u$

$\Rightarrow \quad \sin i = \frac{3}{5} \times \frac{4}{3} \Rightarrow \sin i = \frac{4}{5}$

$\Rightarrow \quad i = 53°$

13.(D) $\Delta Q = \frac{\Delta\phi}{R} = \frac{A(\Delta B)}{R} = \frac{8 \times 10}{16} = 5C$

$\Delta Q = 5C$

14.(C) $g = \frac{GM}{R^2}, \qquad g_p = \frac{G(81M)}{(R/2)^2} = 81 \times 4(g)$

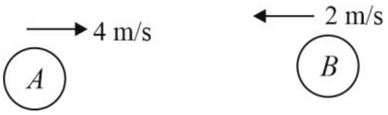
$$T = 2\pi\sqrt{\frac{L}{g}}, \quad T_p = 2\pi\sqrt{\frac{L}{g_p}}$$

$$T_p = 2\pi\sqrt{\frac{L}{81\times 4g}} = \frac{1}{18}2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow \quad T_p = \frac{1}{18}\left(2\pi\sqrt{\frac{L}{g}}\right) \quad \Rightarrow \quad T_p = \frac{T}{18} = \frac{4}{18}s = \frac{2}{9}s$$

15.(B)

16.(C)



$$5\times 4 - 2\times 2 = 5V_1 + 2V_2\,; \qquad e = \frac{1}{2} \Rightarrow \frac{V_2 - V_1}{4-(-2)} = \frac{1}{2}$$

$$\Rightarrow \quad V_2 - V_1 = 3 \quad \ldots \text{(i)}$$

$$2V_2 + 5V_1 = 16 \quad \ldots \text{(ii)}$$

$$\Rightarrow \quad 2V_2 + 5V_1 = 16$$

$$\underline{2V_2 - 2V_1 = 6}$$

$$7V_1 = 10$$

$$\Rightarrow \quad V_1 = \frac{10}{7}\text{m/s} \quad \text{and } V_2 = \frac{31}{7}\text{m/s}$$

17.(C) $m\sqrt{\left(\frac{V^2}{r}\right)^2 + a_t^{\,2}} = \mu mg\,; \qquad V = 57.6\,km/h = 16\,m/s$

$$\Rightarrow \quad \left\{\frac{(16)^2}{160}\times 3\right\}^2 + a_t^{\,2} = \left(\frac{4}{5}\times 10\right)^2 \quad \Rightarrow \quad a_t^{\,2} = 64 - 23.04$$

$$\Rightarrow \quad a_t = \sqrt{40.96} = 6.4\,m/s^2 \quad \Rightarrow \quad a_t = 6.4\,m/s^2$$

18.(D) For no deflection, $q\vec{E} + q(\vec{v}\times\vec{B}) = 0 \quad \Rightarrow \quad \vec{E} = -(\vec{v}\times\vec{B})$

$\Rightarrow \quad \vec{v}\times\vec{B}$ is antiparallel to $\vec{E}$

19.(B) In this case least count = 1MSD – 1VSD

$$= 1\text{mm} - \frac{8}{10}mm = 0.02cm$$

20.(B) $\frac{1}{2}m_{N_2}v_{N_2}^2 = \frac{1}{2}m_{He}\,v_{He}^2$ at 300 K

$$\Rightarrow \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{m_{He}}{m_{N_2}}} \quad \Rightarrow \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{4}{28}} = \sqrt{\frac{1}{7}}$$

SECTION-2

1.(3) $\left(\frac{\text{Stress}}{\text{Strain}}\right)_A = Y_A = \tan 37° = \frac{3}{4}$

$\left(\frac{\text{Stress}}{\text{Strain}}\right)_B = Y_B = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{Y_A}{Y_B} = \frac{3}{4} \times \frac{\sqrt{3}}{1} \quad \Rightarrow \quad \frac{3\sqrt{3}}{4} = \frac{Y_A}{Y_B}$

2.(13) $u_x = 4$, $a_x = -2$ particle starts moving parallel to y-axis when $V_x = 0$

$\Rightarrow \quad v_x = u_x - at \quad \Rightarrow \quad 0 = 4 - 2t \quad \Rightarrow \quad t = 2s$

$x = 4 \times 2 - \frac{1}{2}(2)(2)^2 \quad \Rightarrow \quad x = 4m$

$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad y = \frac{1}{2}(3)(2)^2 \quad \Rightarrow \quad y = 6m$

Displacement $= \sqrt{4^2 + 6^2} = 2\sqrt{13}\,m \quad \Rightarrow \quad x = 13$

3.(12) $B_P = B_1 + B_2 + B_3 \quad = \frac{\mu_0 i}{2\pi d} + \frac{\mu_0 i}{2d} + \frac{\mu_0 i}{2\pi d} = \frac{\mu_0 i}{2d}\left(1 + \frac{2}{\pi}\right)$

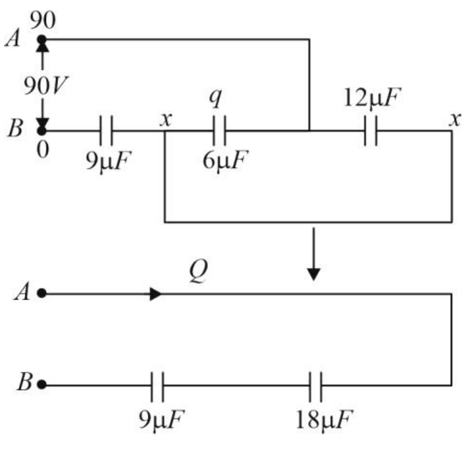
$\Rightarrow \quad B_P = \frac{\mu_0}{10 \times 10^{-2}}\left(\frac{\pi + 2}{\pi}\right) \quad \Rightarrow \quad B_P = 10 \times \frac{4\pi \times 10^{-7}}{\pi}(\pi + 2)$

$\Rightarrow \quad B_P = 40 \times 10^{-7}\left(\frac{22}{7} + 2\right) \quad \Rightarrow \quad B_P = 4 \times 10^{-6}\left(\frac{36}{7}\right)$

$\Rightarrow \quad B_P = 144 \times \frac{10^{-6}}{7} \quad \Rightarrow \quad B_P = \frac{12x}{7} \times 10^{-6} T$

$x = 12$

4.(180) $6\mu F$ and $12\mu F$ are in parallel.



$C_{eq} = \frac{18 \times 9}{18 + 9} = \frac{18 \times 9}{27} = 6\mu F$

$Q = C_{eq}V = 6 \times 90 = 540\,\mu C$

$Q = 540\,\mu C$

ΔV across $18\mu F$ is $90 \times \frac{9}{27} = 30V$

So, charge across $6\mu F$ is $q = CV$

$\Rightarrow \quad q = 6\mu F \times 30V = 180\mu C$

5.(29) $\frac{3}{2V_1} - \frac{1}{-\infty} = \frac{1}{2(20)}$

$\Rightarrow \quad \frac{3}{2V_1} = \frac{1}{2(20)} \quad \Rightarrow V_1 = 60\,cm$

$\frac{1}{V} - \frac{3}{2(56)} = \frac{\frac{-1}{2}}{-20}$

$\Rightarrow \quad \frac{1}{V} - \frac{1}{40} = \frac{3}{112} = \frac{14+15}{560} \qquad \Rightarrow \quad \frac{1}{V} = \frac{29}{560}$

$\Rightarrow \quad V = \frac{560}{29} cm$; $\qquad x = 29$

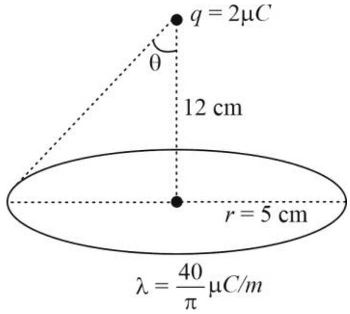
6.(2) $\varepsilon = NBA\omega \Rightarrow \quad \varepsilon = 100 \times \frac{1}{10} \times \left(\frac{1}{10}\right) \times \left(\frac{1}{\pi}\right) \times 2\pi = 2V$

7.(3) $I_{xx'} = \frac{2ML^2}{12} + 2M\left(\frac{L}{2}\right)^2$

$= \frac{2ML^2}{12} + \frac{2ML^2}{4} \Rightarrow I_{xx'} = \frac{4ML^2}{6}$

$\Rightarrow \quad I_{xx'} = \frac{2ML^2}{3} \qquad \Rightarrow \quad x = 3$

8.(100) Force between point charge and ring is $F_e = \frac{KQx}{(R^2+x^2)^{3/2}} \times q$



$F_e = \frac{K \times (\lambda 2\pi r) \times q \times x}{(R^2+x^2)^{3/2}}$

$\Rightarrow \quad F_e = \frac{9 \times 10^9 \times \frac{40}{\pi} \times 10^{-6} \times 2\pi \times 5 \times 10^{-2} \times 2 \times 10^{-6} \times 12 \times 10^{-2}}{(13 \times 10^{-2})^3}$

For equilibrium $mg + F_e = N$

$N = \frac{8260}{(13)^3} + \frac{8640}{(13)^3}$; $\qquad N = \frac{16900}{(13)^3} = \frac{100}{13}$

9.(20) $v = \omega\sqrt{A^2 - x^2}$

When the particle has covered phase equal to $\frac{\pi}{6}$ radians w.r.t. mean position,

$x = A\sin\frac{\pi}{6} \Rightarrow x = \frac{A}{2}. \qquad \Rightarrow \quad v = \omega\sqrt{A^2 - \frac{A^2}{4}}$

$\Rightarrow \quad v = \frac{\sqrt{3}}{2} A\omega \quad \Rightarrow \quad v = \frac{\sqrt{3}}{2} v_{\max}$

$\Rightarrow \quad v = \frac{\sqrt{3}}{2} \times \frac{1}{10} \quad \Rightarrow \quad v = \frac{\sqrt{3}}{20}$ m/s $\quad \Rightarrow \quad x = 20$

10.(5) The mass defect $\Delta m = Zm_H + Nm_n - M_{Sn}$

$= (50 \times 1.0078 + 70 \times 1.0086 - 119.9022)$

$= (50.39 + 70.602 - 119.9022)$

$= 1.0898u$

Binding energy $= \Delta mc^2$

$= 1.0898 \times 931$

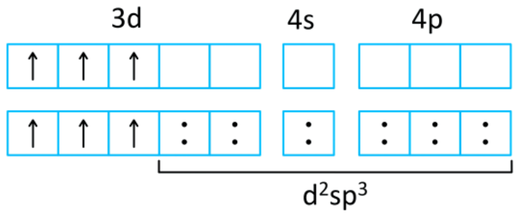
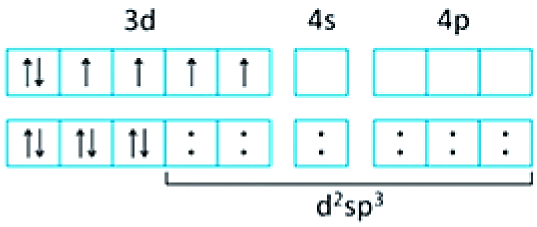
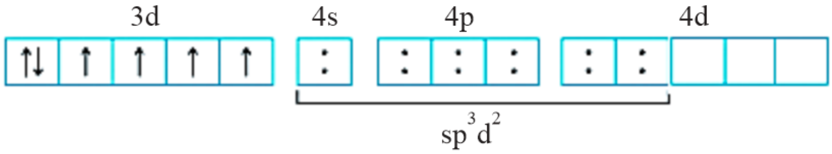
$= 1014.6038 \quad \simeq 203 \times 5 MeV$

CHEMISTRY

SECTION-1

1.(A) $\underset{Y^{2+}}{Pb^{2+}} + \underset{X}{KI} \rightarrow \underset{(yellow)Z}{PbI_2\downarrow}$

2.(C) $P : n = 0$
$Q : n = 4$
$R : n = 3$

$R \rightarrow$	$Cr^{3+}(d^3)$	
	$[Cr(NH_3)_6]^{3+}$	
$P \rightarrow$	$Fe^{2+}(d^6)$	
	$[Fe(CN)_6]^{4-}$	
$Q \rightarrow$	$[Fe(H_2O)_6]^{2+}$	

3.(B) Electronic configuration of Ytterbium = $[Xe]4f^{14}6s^2$.

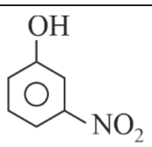
Most common oxidation state of [Lanthanoids] is +3 thus $Y^{3+} = [Xe]4f^{13}$.

4.(B) NH_4OH is relatively stronger base than acid HCN.

5.(C) Ortho nitrophenol is involved in intra molecular hydrogen bonding and meta & para isomers of nitrophenols involved in intermolecular hydrogen bonding with the other molecules.

6.(A) Beryllium is not chemically inert and form compounds easily.

7.(C) Intermolecular H-bonding occurs in:

Acetic acid	CH_3COOH
m-nitrophenol	
ortho-boric acid	H_3BO_3

8.(B) $\mu = \sqrt{n(n+2)}$

$Ni^{2+} = 3d^8; n = 2$

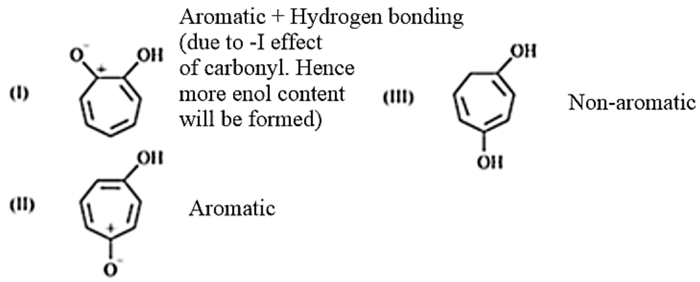
9.(B) Between two polypeptide chains disulphide linkage is present.

10.(B) Due to small size of boron, its ionization to +3 state is difficult, thus it forms covalent compounds.

11.(A) Most stable carbocation is formed in option **(A).**

12.(B) More electronegative atom is attached therefore more stable anion (conjugate ion) is formed. Hence more acidic in nature.

13.(A)



14.(D)

15.(D) In 3rd compound,

–R effect of NO_2 is present.

In 1st compound, –I effect of NO_2 is present.

In 4th compound, only resonance is present.

16.(D) In option (A) and (C) lone pair is involved in resonance, hence unavailable. Secondary aliphatic amine is more basic than ammonia.

17.(C) Mole fraction of solvent decreases means mole fraction of solute increases.

18.(C) $Ba(NO_3)_2 + K_2CrO_4 \rightarrow \underset{\substack{\text{yellow ppt.} \\ (A)}}{BaCrO_4} \downarrow$

$Ba(NO_3)_2 \xrightarrow{\text{dil. } H_2SO_4} \underset{\substack{\text{white ppt.} \\ (B)}}{BaSO_4} \downarrow$

19.(B)

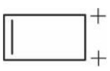
20.(C) Cesium $\rightarrow$ positive & zero

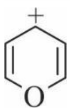
Fluorine $\rightarrow$ Negative & zero

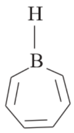
Iodine $\rightarrow$ Positive, Negative & zero

Xenon $\rightarrow$ Positive & zero

SECTION-2

1.(3)  has 2π electrons and cyclic resonance, hence aromatic.

 has 6π electrons and cyclic resonance, hence aromatic.

 has 6π electrons and cyclic resonance, hence aromatic.

2.(6) $3Fe + 4H_2O \longrightarrow Fe_3O_4 + 4H_2$

moles of $H_2O = \frac{18}{18} = 1$

moles of Fe required $= \frac{3}{4}$

mass of Fe required $= \frac{3}{4} \times 56$

$$\frac{W}{7} = \frac{3}{4} \times \frac{56}{7} = 6$$

3.(2) Anode : $2Cl^- \longrightarrow Cl_2 + 2e^-$

moles of Cl_2 formed $= \frac{1}{2} \times 4 = 2$

4.(4)

$$\begin{array}{l} CH_3-CH_2-\underset{\displaystyle CH_3}{\underset{|}{C}} = \underset{\displaystyle CH_3}{\underset{|}{C}}-CH_2-CH_3 + HBr \xrightarrow{\text{peroxide}} \quad A \end{array}$$

$$CH_3-CH_2-\underset{\displaystyle CH_3}{\underset{|}{\overset{*}{C}H}} - \overset{\displaystyle Br}{\overset{|}{\underset{\displaystyle CH_3}{\underset{|}{C^*}}}}-CH_2-CH_3$$

(A)

Number of chiral centres = 2

Number of stereoisomers = 2^2 = 4

5.(4) $_{18}Ar = 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6$

number of electrons having m_l = 1, are 4.

6.(4) Oxidation state of S in SO_3, $H_2S_2O_7$, $H_2S_2O_8$ and SO_2Cl_2 is + 6

7.(5) Bond order of N_2^+ = 2.5

bond order of $O_2^+ = 2.5$

sum = 5

8.(2) Let order with respect to A = x

order with respect to B = y

rate = k $[A]^x [B]^y$

$2 \times 10^{-4} = k(1 \times 10^{-2})^x (2 \times 10^{-2})^y$ ...(i)

$4 \times 10^{-4} = k(2 \times 10^{-2})^x (2 \times 10^{-2})^y$ ...(ii)

$8 \times 10^{-4} = k(2 \times 10^{-2})^x (4 \times 10^{-2})^y$ ...(iii)

from (i)/(ii) $\Rightarrow x = 1$

from (ii)/(iii) $\Rightarrow y = 1$

order of reaction $= x + y = 1 + 1 = 2$

9.(5) $-OC_2H_5, -NHCH_3, -OH, -Br, -CH_3$ are ortho and para directors.

10.(6)

$q = 10.04\, kJ$

$W = -P_{ext} \times (V_2 - V_1) = -2 \times (40 - 20)$

$W = -40\ L-atm$

$W = -40 \times 101 J$

$W = -4040\ J$

$W = -4.04\, kJ$

$\Delta U = q + W \Rightarrow 10.04 - 4.04 \Rightarrow \Delta U = 6\ kJ$

MATHEMATICS

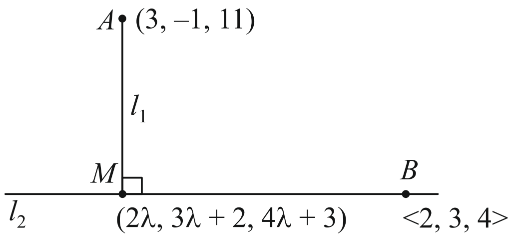
SECTION-1

1.(C) $f(0^-) = e^a$

$f(0) = b$

$f(0^+) = \frac{2}{3} \Rightarrow b = e^a = \frac{2}{3}$

2.(A)



Direction ratios of AM is $< 2\lambda - 3, 3\lambda + 3, 4\lambda - 8 >$

As $l_1 \perp l_2$

$\Rightarrow \quad 2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0$

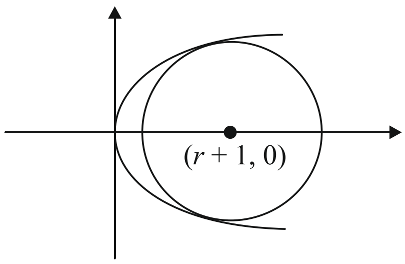
$\Rightarrow \quad \lambda(4 + 9 + 16) - 6 + 9 - 32 = 0$

$\Rightarrow \quad \lambda = 1$

$M(2, 5, 7)$

Dr's of AM are $< -1, 6, -4 >$

3.(B)



$(x - r - 1)^2 + y^2 = r^2$

$\Rightarrow \quad (x - r - 1)^2 + 4x = r^2$

$\Rightarrow \quad x^2 + (4 - (2r + 2))x + 2r + 1 = 0$

$D = 0 \Rightarrow r = 4$

4.(A) Given $x_1 + y_1 = -8 \quad \Rightarrow \quad y_1 = -8 - x_1;$

$x_2 + y_2 = -8 \quad \Rightarrow \quad y_2 = -8 - x_2$

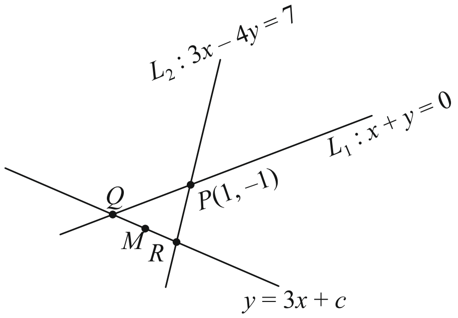
and $x_3 + y_3 = -8 \quad \Rightarrow \quad y_3 = -8 - x_3$

now check

$\begin{vmatrix} x_1 & -8 - x_1 & 1 \\ x_2 & -8 - x_2 & 1 \\ x_3 & -8 - x_3 & 1 \end{vmatrix}$ value of this determinant comes out to be zero.

$\therefore$ Points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear.

5.(C)



We get, $Q \equiv \left(-\frac{c}{4}, \frac{c}{4}\right)$, $R \equiv \left(\frac{7+4c}{-9}, \frac{7+c}{-3}\right)$

Let $M = (h, k)$

Then, $2h = \frac{-7}{9} - \frac{25}{36}c$, $2k = \frac{-7}{3} + \frac{c}{4} - \frac{c}{3}$

$\Rightarrow \quad 3h - 25k = 28$

Therefore, required locus $3x - 25y = 28$

6.(A) Let $u = \frac{ax_i + b}{p}$ $\quad \therefore \quad \bar{u} = \frac{a\bar{x} + b}{p}$

$$\therefore \quad SD = \sqrt{\frac{\sum (u - \bar{u})^2}{\sum f}} = \sqrt{\frac{\sum\left(\frac{ax_i + b}{p} - \frac{a\bar{x} + b}{p}\right)^2}{\sum f}}$$

$$= \sqrt{\frac{\sum\left(\frac{ax_i + b - a\bar{x} - b}{p}\right)^2}{\sum f}} = \sqrt{\frac{a^2}{p^2}\frac{\sum\left(x_i - \bar{x}\right)^2}{\sum f}} = \sqrt{\frac{a^2}{p^2}\sigma_x^2} = \left|\frac{a}{p}\right|\sigma_x$$

7.(C) $(x+1)^{35}\ (1-x)^{35} = (1-x^2)^{35}$

$\Rightarrow \quad ({}^{35}C_0 x^{35} + {}^{35}C_1 x^{34} + ... + {}^{35}C_{35})$

$\quad ({}^{35}C_0 - {}^{35}C_1 x + {}^{35}C_2 x^2 .... - {}^{35}C_{35} x^{35}) = (1-x^2)^{35}$

$\Rightarrow \quad -{}^{35}C_0 \cdot {}^{35}C_{15} + {}^{35}C_1 \cdot {}^{35}C_{16} - ..... - {}^{35}C_{20}\ {}^{35}C_{35} =$

Co-efficient of x^{50} in $(1-x^2)^{35} = -{}^{35}C_{25}$

$\therefore \quad {}^{35}C_0 \cdot {}^{35}C_{15} - {}^{35}C_1 \cdot {}^{35}C_{16} + ... + {}^{35}C_{20} \cdot {}^{35}C_{35} = {}^{35}C_{25}$

8.(C) a, b, c are in AP $\Rightarrow \quad a + c = 2b \quad$ ….(1)

$(b-a)$, $(c-b)$, a are in GP

$\therefore \quad (c-b)^2 = (b-a)a$

$\quad (2b - a - b)^2 = (b-a)a \quad$ [from (1)]

$\Rightarrow \quad (b-a)^2 = a(b-a)$

$\Rightarrow \quad \therefore b - a = a \quad (\because b \neq a)$

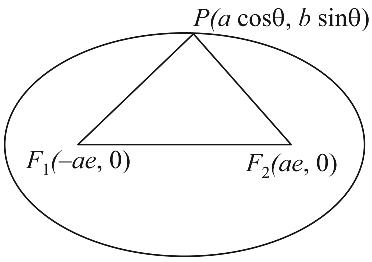
$\quad b = 2a \quad$ …..(2)

Again from (1) and (2) we get

$a + c = 4a \quad c = 3a$

$a : b : c = a : 2a : 3a = 1 : 2 : 3$

9.(A)



$PF_1 = a(1+e\cos\theta)$

$PF_2 = a(1-e\cos\theta)$

$F_1F_2 = 2ae$

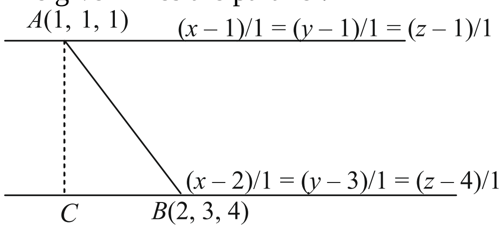
In centre (I) of $\Delta PF_1F_2 \equiv \left(ae\cos\theta, \dfrac{be}{1+e}\sin\theta\right)$

The locus of incentre of triangle PF_1F_2 is an ellipse whose eccentricity is $\sqrt{\dfrac{2e}{1+e}}$, where $e = \dfrac{1}{2}$ is the eccentricity of the given ellipse.

$\therefore \quad \sqrt{\dfrac{2e}{1+e}} = \sqrt{\dfrac{2}{3}}$

$\Rightarrow \quad 81e^2 = 54$

10.(C) The given lines are parallel.



From the figure, we get $BC = \dfrac{(2-1)1}{\sqrt{3}} + \dfrac{(3-1)1}{\sqrt{3}} + \dfrac{(4-1)1}{\sqrt{3}}$

$= \dfrac{1+2+3}{\sqrt{3}} = 2\sqrt{3} \qquad \left(\because BC = \left|\dfrac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}|}\right| \text{ where } \vec{n} \text{ is vector parallel to line } BC\right)$

$AB = \sqrt{1+4+9} = \sqrt{14}$

Shortest distance $= AC = \sqrt{14-12} = \sqrt{2}$

11.(C) $\because \quad (x, y) \in R \Rightarrow x^y = y^x$

$\therefore \quad (x, x) \in R$ as $x^x = x^x \ \forall\, x \in I - \{0\}$

$\therefore \quad R$ is reflexive

Now $\quad (x, y) \in R \Rightarrow x^y = y^x \Rightarrow y^x = x^y$

$\Rightarrow \quad (y, x) \in R \qquad \therefore R$ is symmetric

12.(C) $$\lim_{x\to 0} \frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x - x + 1}\right)}{x^2+2\sin x - \sin^2 x + x}$$

$$\lim_{x\to 0} \frac{(1+2\frac{\sin x}{x})\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x - x + 1}\right)}{x+2\frac{\sin x}{x} - \frac{\sin^2 x}{x} + 1} = \frac{3\times 2}{3} = 2$$

13.(B) $I=\int_{-1}^{1}\frac{dx}{(1+e^x)(1+x^2)}$ ....(1)

$I=\int_{-1}^{1}\frac{dx}{(1+e^{-x})(1+x^2)}$ ...(2)

$2I=\int_{-1}^{1}\frac{1}{1+x^2}\left(\frac{1}{1+e^x}+\frac{1}{1+e^{-x}}\right)$

$2I=\int_{-1}^{1}\frac{dx}{1+x^2}$

$2I=2\int_{0}^{1}\frac{dx}{1+x^2}$

$I=(\tan^{-1}x)_0^1=\frac{\pi}{4}$

14.(C) We have $|A|=\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$ $[C_1\to C_1+C_2+C_3]$

$=\begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}=(3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$

$=(3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}=\lambda^2(3x+\lambda)$

[Take $3x+\lambda$ common and use $R_2\to R_2-R_1, R_3\to R_3-R_1$]

Thus, A^{-1} will exist if $\lambda\neq 0$ and $3x+\lambda\neq 0$.

15.(B) As $f(\alpha)=f(\beta)=f(\gamma)=0$

$\Rightarrow f(x)$ is not one-one.

If $y=(x-\alpha)(x-\beta)(x-\gamma)$

Any odd degree polynomial is always onto

16.(C) $\because$ $|\vec{a}+2\vec{b}+3\vec{c}|=\sqrt{3+2\sqrt{2}}$

$\therefore$ $4\cos\alpha+6\cos\beta=-11+2\sqrt{2}-12\cos\theta$

The value of $4\cos\alpha+6\cos\beta$ will be maximum when $\theta=\frac{2\pi}{3}$ i.e., $\cos\theta=-\frac{1}{2}$

$\therefore$ The maximum value of $4\cos\alpha+6\cos\beta$ is $2\sqrt{2}-5$

17.(B) Let $(2x^2-3x+1)^{11}=a_0+a_1x+a_2x^2+...+a_{22}x^{22}$;

To find $a_0+a_2+a_4+...+a_{22}$?

$S_E+S_O=P(1)=0$ where $P(x)=(2x^2-3x+1)^{11}$ ...(1)

$S_E-S_O=P(-1)=6^{11}$ ....(2)

$\Rightarrow$ $2S_E=6^{11}$ $\Rightarrow$ $S_E=3\cdot 6^{10}$

18.(A) Point of intersection of $7x+2y-9=0$ and $x-7y+6=0$ is (1, 1)

Required equation of circle is $(x-2)^2+(y-1)^2=1^2$

19.(B) $\because \quad \frac{dy}{dx}+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}$

$\Rightarrow \quad \frac{dy}{dx}=\sin\frac{x-y}{2}-\sin\frac{x+y}{2}$

$\Rightarrow \quad \frac{dy}{dx}=-2\cos\frac{x}{2}\cdot\sin\frac{y}{2}$

$\Rightarrow \quad \int \operatorname{cosec}\frac{y}{2}\,dy=-2\int\cos\frac{x}{2}\,dx$

$\Rightarrow \quad 2\log\tan\frac{y}{4}=-2\cdot2\sin\frac{x}{2}+c'$

$\therefore \quad \log\left(\tan\frac{y}{4}\right)=c-2\sin\frac{x}{2}$

20.(A) $|z-1|+|z-4\sin\theta|=\sqrt{5}$ r

For ellipse, $|4\sin\theta-1|<\sqrt{5}$

$\Rightarrow \quad \frac{1-\sqrt{5}}{4}<\sin\theta<\frac{1+\sqrt{5}}{4}$

$\therefore \quad -\sin\frac{\pi}{10}<\sin\theta<\sin\frac{3\pi}{10}$

SECTION-2

1.(3) $3^{\left(\frac{1}{4}+2\left(\frac{1}{8}\right)+3\left(\frac{1}{16}\right)+\ldots\infty\right)}=?$

Let $x=\frac{1}{4}+2\left(\frac{1}{8}\right)+3\left(\frac{1}{16}\right)+4\left(\frac{1}{32}\right)+\ldots+\infty\ (A.G.P.)$

$\frac{1}{2}\cdot x=\frac{1}{8}+2\left(\frac{1}{16}\right)+3\left(\frac{1}{32}\right)+\ldots\infty$

$\Rightarrow \quad x-\frac{x}{2}=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\infty$

$\Rightarrow \quad \frac{x}{2}=\frac{\frac{1}{4}}{1-\frac{1}{2}}$

$\Rightarrow \quad \frac{x}{2}=\frac{1}{4}\times\frac{2}{1}=\frac{1}{2} \qquad \Rightarrow \qquad x=1$

$\therefore \quad 3^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}}+\ldots\infty=3$

2.(6) $x^2+x+1=0$

$x=\omega,\omega^2$

Now, $x+\frac{1}{x}=\omega+\omega^2=-1$

$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=1-2=-1$

$x^3+\frac{1}{x^3}=1+1=2$

$x^4+\frac{1}{x^4}=\left(x^2+\frac{1}{x^2}\right)^2-2=1-2=-1$

$x^5+\frac{1}{x^5}=\left(x^4+\frac{1}{x^4}\right)\left(x+\frac{1}{x}\right)-\left(x^3+\frac{1}{x^3}\right)$

$=1-2=-1$

$x^6+\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)^2-2=4-2=2$

Hence, the value of required expression $=\frac{1}{9}(9\times 4+18)=\frac{1}{9}\times 54=6$

3.(6) p = probability of drawing a white ball $=\frac{16}{24}=\frac{2}{3}$

q = probability of drawing a black ball $=\frac{8}{24}=\frac{1}{3}$

Probability of drawing a white ball 4^{th} time on the 7^{th} draw.
= Probability of drawing 3 white balls in first 6 draws × probability of drawing a white ball at the 7^{th} draw.

$=\left({}^6C_3p^3q^3\right)\times\frac{16}{24}=\left(20\times\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3\right)\times\frac{2}{3}=\left(\frac{2}{3}\right)^3\times\frac{40}{81}$

Clearly a = 2, b = 3.
Hence, the value of $a+b+1=$ is $2+3+1=6$

4.(1) $\frac{dy}{dx}-y=1-e^{-x}, I.F.=e^{-x}$

$\therefore \quad y\cdot e^{-x}=\int(e^{-x}-e^{-2x})\,dx$

$y\cdot e^{-x}=-e^{-x}+\frac{1}{2}e^{-2x}+C$

$y=-1+\frac{1}{2}e^{-x}+C.e^{x}$

$\Rightarrow \quad y(0)=-1+\frac{1}{2}+C=\frac{-1}{2}$

$\Rightarrow \quad C=0$

$\lim_{x\to\infty}|y|=\lim_{x\to\infty}\left|-1+\frac{1}{2}e^{-x}\right|=1$

5.(3) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}f(x)\left(\frac{f(h)-1}{h}\right)=f(x)\cdot f'(0)=3f(x)$

$f(x)=ke^{3x}$

Since $f(0)=1, \quad \Rightarrow \quad k=1$

$\Rightarrow \quad f(x)=e^{3x}$

$f'(x)=3e^{3x}$

$\Rightarrow \quad f'(15)=3e^{45}$

6.(2) Given $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$

$$\vec{b}\cdot\vec{c} = \vec{b}\cdot\{(2\vec{a}\times\vec{b}) - 3\vec{b}\}$$

$$\vec{b}\cdot\vec{c} = -3|\vec{b}|^2 \qquad \ldots(i)$$

Now $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$

$$= 16 - 4 = 12$$

And $|\vec{c}|^2 = |(2\vec{a}\times\vec{b}) - 3\vec{b}|^2$

$$|\vec{c}|^2 = 4|\vec{a}\times\vec{b}|^2 + 9\left|\vec{b}\right|^2 - \underbrace{6\vec{b}.(2\vec{a}\times\vec{b})}_{zero}$$

$$= 48 + 144 = 192$$

$$|\vec{c}| = 8\sqrt{3}$$

Now, $\cos\theta = \dfrac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|} = \dfrac{-3|\vec{b}|^2}{|\vec{b}||\vec{c}|} = \dfrac{-3|\vec{b}|}{|\vec{c}|} = \dfrac{-3(4)}{8\sqrt{3}}$

$$= \frac{-\sqrt{3}}{2}$$

Hence $\theta = \dfrac{5\pi}{6}$ $\qquad \therefore [\theta] = 2$

7.(1) If $e^{-\int\frac{x}{1+x}dx} = e^{-\int\frac{x+1-1}{x+1}dx} = e^{-x+\ell n|(x+1)|} = e^{-x}\cdot(x+1)$

$$y\cdot(x+1)e^{-x} = \int\frac{x+1}{x+1}e^{-x}dx + c$$

$$y\cdot(x+1)\,e^{-x} = -e^{-x} + c$$

$$\Rightarrow \quad y = -\frac{1}{x+1} + \frac{ce^x}{x+1}$$

$$x = 0,\ y = -1 \ \Rightarrow\ -1 = -1 + c \ \Rightarrow\ c = 0$$

$$\Rightarrow \quad y = -\frac{1}{1+x}$$

$$y(2) = -\frac{1}{3}$$

8.(0) We have $g(x) = f\left(\dfrac{x}{f(x)}\right)$

On differentiating w.r.t. x we get

$$g'(x) = f'\left(\frac{x}{f(x)}\right)\times\left(\frac{f(x) - xf'(x)}{f^2(x)}\right)$$

$$\therefore \quad f'(1) = f'\left(\frac{1}{f(1)}\right)\times\left(\frac{f(1) - f'(1)}{f^2(1)}\right)$$

As $f(1) = f'(1)$

$$\Rightarrow \quad g'(1) = 0$$

9.(4) $B = A\left(I + 4A + 6A^2 + 4A^3 + A^4\right) = A\left(I + A\right)^4$

$|B| = |A||I + A|^4 = 1$ (Given) (As $|A| = 1$)

$\Rightarrow \quad \begin{vmatrix} 1+\cos\alpha & \sin\alpha \\ -\sin\alpha & 1+\cos\alpha \end{vmatrix}^4 = 1 \qquad \Rightarrow \quad [(1+\cos\alpha)^2 + \sin^2\alpha]^4 = 1$

$\Rightarrow \quad 16(1+\cos\alpha)^4 = 1 \qquad \Rightarrow \quad 1+\cos\alpha = \frac{1}{2}$

$\Rightarrow \quad \cos\alpha = -\frac{1}{2}$

$\therefore$ Number of values of α in $[-2\pi, 2\pi]$ are 4.

10.(32) Let $\frac{\pi}{31} = \theta$, then

$$\cos\theta\cos 2\theta\cos 4\theta\cos 8\theta\cos 16\theta = \frac{(2\sin\theta\sin\theta)\cos 2\theta\cos 4\theta\cos 8\theta\cos 16\theta}{2\sin\theta}$$

$$= \frac{(2\sin 2\theta\cos 2\theta)\cos 4\theta\cos 8\theta\cos 16\theta}{4\sin\theta} = \frac{(2\sin 4\theta\cos 4\theta)\cos 8\theta\cos 16\theta}{8\sin\theta}$$

$$= \frac{(2\sin 8\theta\cos 8\theta)\cos 16\theta}{16\sin\theta} = \frac{2\sin 16\theta\cos 16\theta}{32\sin\theta} = \frac{\sin 32\theta}{32\sin\theta} = \frac{\sin(\pi+\theta)}{32\sin\theta} = \frac{-1}{32}$$